

Exercise 35

Find the limit or show that it does not exist.

$$\lim_{x \rightarrow \infty} \arctan(e^x)$$

Solution

Make the substitution, $u = e^x$. Then as $x \rightarrow \infty$, $u \rightarrow \infty$.

$$\lim_{x \rightarrow \infty} \arctan(e^x) = \lim_{u \rightarrow \infty} \arctan u$$

Make another substitution, $v = \arctan u$, or $\tan v = u$. Then as $u \rightarrow \infty$, $v \rightarrow \frac{\pi}{2}^-$.

$$\lim_{x \rightarrow \infty} \arctan(e^x) = \lim_{v \rightarrow \frac{\pi}{2}^-} v$$

$$\lim_{x \rightarrow \infty} \arctan(e^x) = \frac{\pi}{2}$$